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Misinterpretations of the diabatic regenerator performances

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Abstract—This paper presents sources of discrepancy between 'hot' (ε_h) and 'cold' (ε_c) effectiveness of a diabatic regenerator. The regenerator wall finite conductivity model is used to derive the difference in ε_h and ε_c. A need for experimental measurement of average external insulation surface temperatures in both periods is stressed.

INTRODUCTION

MUCH HAS been published concerning the differences that have been found between the experimental results and theoretical predictions of the regenerator performance at the cyclic steady state. One of the facts that is apparent in the work of many authors stands together with the wrong understanding and explanation of the diabatic regenerator effectiveness. The presence of the finite heat loss from the regenerator operating above the ambient temperature, and the finite heat gain from the surroundings to the regenerator operating at low temperatures, is always notable even with an insulation being employed. Bretherton [1] has found experimentally from the cyclic data of the cryogenic regenerators that the 'cold effectiveness' (ε_c) is always greater than the 'hot effectiveness' (ε_h). The converse is true ($\varepsilon_h > \varepsilon_c$) for regenerators operating at high temperatures, since the regenerator wall continues to lose heat to the surroundings during both the hot and cold gas flow period. Experimental investigations on regenerators performed by Ajitsaria [2], MacDonald [3], Rizvi [4], Hollins [5], Mitchell [6], Hargraves *et al.* [7], as well as some computer simulations developed by Amooie-Foumeny [8] and some of the above authors, have utilized the terms 'hot' and 'cold effectiveness' in evaluating the regenerator performance. What all these authors mean by 'hot effectiveness' is simply the definition of an adiabatic regenerator effectiveness in terms of the hot period parameters

$$\varepsilon_h = \frac{(\Pi/\Lambda)_h}{(\Pi/\Lambda)_{min}} \frac{T_{h,in} - T_{h,out}}{T_{h,in} - T_{c,in}} \tag{I}$$

while, by the 'cold effectiveness' the following expression is meant:

$$\varepsilon_c = \frac{(\Pi/\Lambda)_c}{(\Pi/\Lambda)_{min}} \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} \tag{II}$$

which is again the definition of an adiabatic regenerator effectiveness but in terms of the cold period parameters. Furthermore, some of the authors consider expressions (I) and (II) as being valid just for balanced $[(\Pi/\Lambda)_h = (\Pi/\Lambda)_c]$ regenerators, so that one can find just $\varepsilon_h = (T_{h,in} - T_{h,out})/(T_{h,in} - T_{c,in})$ and $\varepsilon_c = (T_{c,out} - T_{c,in})/(T_{h,in} - T_{c,in})$ in use.

Naturally, for a diabatic regenerator, ε_h and ε_c, as defined by equations (I) and (II), respectively, can never yield the same value since these parameters are by no means the measures of actual heat being transferred in the regenerator at cyclic steady states. With all other assumptions being the same, an equality $\varepsilon_h = \varepsilon_c$ can be true just for an adiabatic regenerator. Not being aware of the misuse of equations (I) and (II), some of the mentioned authors introduced an average effectiveness, loosely defined as $\varepsilon_{ave} = (\varepsilon_h + \varepsilon_c)/2$, as an actual measure of the diabatic regenerator performance. Within the experimental and computational accuracy they observed a rather good agreement between the experimental and re-simulated values of ε_{ave}. This was simply a coincidence since only nearly balanced regenerators have been considered. It must be clear that the actual diabatic regenerator effectiveness ε can coincide with ε_{ave} just in the case when the quantities of heat being exchanged among the regenerator and its surroundings are exactly the same in both hot and cold periods. These quantities of heat cannot be predicted without the knowledge of the average regenerator external surface temperatures in each period and the corresponding external heat transfer coefficients. Unfortunately, the need for measuring the heat losses of diabatic regenerators in order to estimate its actual performance has never been realized by the investigators.

This paper intends to clarify the origin of difference between ε_h and ε_c and to indicate what one has to measure at the diabatic regenerator cyclic equilibrium conditions in order to estimate the effectiveness as a single measure of its performance. Amooie-Foumeny's [8] regenerator wall finite conductivity model is arbitrary chosen to derive the working formulae. Some other diabatic regenerator models, like the one presented in ref. [7], fail to yield a satisfactory description.

NOMENCLATURE

- A packing to gas heat transfer area [m^2]
- A_e insulation external surface area, $2(R+\delta_w+\delta_e)\pi L$ [m^2]
- A_f frontal area of the regenerator, $R^2\pi$ [m^2]
- A_i wall internal surface area, $2R\pi L$ [m^2]
- a_e insulation thermal diffusivity [$m^2 s^{-1}$]
- a_w wall thermal diffusivity [$m^2 s^{-1}$]
- B number of heat loss units, $A_e\alpha_o/(\dot{M}c_p)$ [dimensionless]
- c_p specific heat of gas at constant pressure [$J\,kg^{-1}\,K^{-1}$]
- c_s specific heat of solid packing [$J\,kg^{-1}\,K^{-1}$]
- L bed length [m]
- $\dot{M}$ gas mass flow rate [$kg\,s^{-1}$]
- M^s total mass of packing [kg]
- $\dot{m}$ superficial gas mass velocity [$kg\,m^{-2}\,s^{-1}$]
- n integer (large enough)
- P duration of gas flow period [s]
- Q_{act} actual quantity of heat being transferred in the regenerator [J]
- Q_{max} maximal quantity of heat that can be transferred in the regenerator, defined by equation (26) [J]
- R wall internal radius [m]
- r radial coordinate [m]
- T_c temperature of the cold gas [K]
- T_e temperature of the external insulation [K]
- T_h temperature of the hot gas [K]
- T_s temperature of the solid packing [K]
- T_w temperature of the wall [K]
- T_∞ temperature of the surroundings [K]
- V bed volume, $A_f L$ [m]
- x distance along the bed (hot gas downstream) [m].

Greek symbols

- α gas to packing heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
- α_i gas to wall heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
- α_o outside heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
- δ_e thickness of the external insulation [m]
- δ_w thickness of the wall [m]
- ε regenerator effectiveness, defined by equation (27) [dimensionless]
- ε_{ave} average effectiveness, $(\varepsilon_h+\varepsilon_c)/2$ [dimensionless]
- ε_c 'cold effectiveness', defined by equation (II) [dimensionless]
- ε_h 'hot effectiveness', defined by equation (I) [dimensionless]
- $\Delta\varepsilon_c$ quantity defined by equation (37) [dimensionless]
- $\Delta\varepsilon_h$ quantity defined by equation (36) [dimensionless]
- Λ dimensionless length for a regenerator, $\alpha A/(\dot{M}c_p)$ [dimensionless]
- λ_e external insulation thermal conductivity [$W\,m^{-1}\,K^{-1}$]
- λ_w wall thermal conductivity [$W\,m^{-1}\,K^{-1}$]
- Π dimensionless period for a regenerator, $\alpha A/(M_s c_s)$ [dimensionless]
- $\bar{\rho}$ bulk density of packing [$kg\,m^{-3}$]
- τ time [s].

Subscripts

- c cold
- e external insulation
- h hot
- in inlet
- out outlet
- surf surface.

Superscript

- space and time average.

LOCAL AND OVERALL HEAT BALANCE EQUATIONS FOR A DIABATIC REGENERATOR

In this section we present how a consistent overall heat balance equation for a diabatic regenerator should be derived from a set of local (micro) balance equations. As for any thermal device, the overall energy equation for a regenerator interrelates, in an algebraic form, the parameters at the system boundary. It provides the information on heat being actually transferred within the system. Applying the energy balance (again the first law of thermodynamics) to an elemental part of a regenerator, the local balance equations are derived. These are usually differential equations describing the actual energy transfer mechanisms within the device according to the basic assumptions.

Once the underlying assumptions are settled the overall balance equation can be written. If there is any doubt of its consistency with the local balances, one can verify it by formal integration of the differential equations over the space and time domain. This procedure will make use of all boundary and initial conditions and will yield the correct integral heat balance equation.

We will describe now this technique on a consistent diabatic regenerator model. For this purpose the fixed bed regenerator wall finite conductivity model is taken from ref. [8]. For the sake of preciseness only minor changes are made in the model. The assumptions

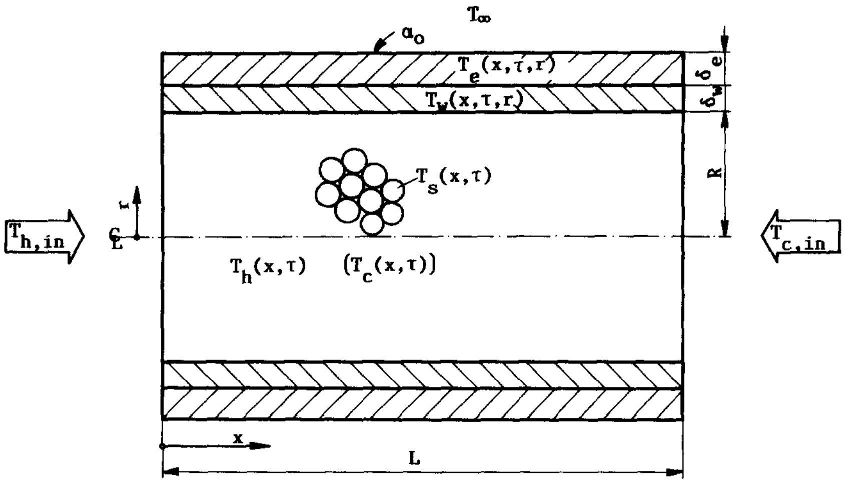

FIG. 1. Basic notation used in the diabatic regenerator model.

underlying the mathematical model are not listed here, but will be self-evident from the forms of the governing equations and boundary conditions.

Referring to Fig. 1 and the Nomenclature, this model of a diabatic counterflow regenerator operation at cyclic steady state is as follows (note that, according to Fig. 1, the regions of the independent spatial coordinates in all equations are: axial coordinate $0 \leqslant x \leqslant L$; radial coordinate $R \leqslant r \leqslant R+\delta_w$ for the wall, and $R+\delta_w \leqslant r \leqslant R+\delta_w+\delta_e$ for the insulation layer).

Hot gas flow periods (*time domain* $n(P_h+P_c) \leqslant \tau \leqslant n(P_h+P_c)+P_h$; $n \gg 1$)

The gas to packing and wall local heat flow equation is

$$(\dot{m}c_p)_h V\frac{\partial T_h(x,\tau)}{\partial x} + \bar{\rho}Vc_s\frac{\partial T_s(x,\tau)}{\partial \tau} + \alpha_{i,h}A_i[T_h(x,\tau)-T_w(x,\tau,R)] = 0 \quad (1)$$

and the hot gas enters the regenerator at

$$T_h(0,\tau) = T_{h,in}. \quad (2)$$

The solid phase (packing) is heated solely by convection. It is lumped according to

$$\bar{\rho}Vc_s\frac{\partial T_s(x,\tau)}{\partial \tau} + \alpha_h A[T_s(x,\tau)-T_h(x,\tau)] = 0 \quad (3)$$

and enters each new cycle at the same temperature distribution

$$T_s[x,n(P_h+P_c)] = T_s[x,(n+1)(P_h+P_c)]. \quad (4)$$

Transient radial heat conduction through the wall is governed by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_w(x,\tau,r)}{\partial r}\right) - \frac{1}{a_w}\frac{\partial T_w(x,\tau,r)}{\partial \tau} = 0. \quad (5)$$

Wall to insulation contact is perfect

$$T_w(x,\tau,R+\delta_w) = T_e(x,\tau,R+\delta_w) \quad (6)$$

and the wall is heated convectively by the hot gas

$$-\lambda_w\frac{\partial T_w(x,\tau,R)}{\partial r} + \alpha_{i,h}[T_w(x,\tau,R)-T_h(x,\tau)] = 0. \quad (7)$$

The wall cyclicly enters into hot periods

$$T_w[x,n(P_h+P_c),r] = T_w[x,(n+1)(P_h+P_c),r]. \quad (8)$$

Heat is conducted through the external insulation layer according to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_e(x,\tau,r)}{\partial r}\right) - \frac{1}{a_e}\frac{\partial T_e(x,\tau,r)}{\partial \tau} = 0 \quad (9)$$

with the incoming heat flux from the wall

$$-\lambda_e\frac{\partial T_e(x,\tau,R+\delta_w)}{\partial r} + \lambda_w\frac{\partial T_w(x,\tau,R+\delta_w)}{\partial r} = 0 \quad (10)$$

and the convective heat flux to the surroundings

$$-\lambda_e\frac{\partial T_e(x,\tau,R+\delta_w+\delta_e)}{\partial r} + \alpha_{o,h}[T_\infty - T_e(x,\tau,R+\delta_w+\delta_e)] = 0 \quad (11)$$

so that the insulation undergoes the cyclic equilibrium as well

$$T_e[x,n(P_h+P_c),r] = T_e[x,(n+1)(P_h+P_c),r]. \quad (12)$$

Cold gas flow periods (*time domain* $n(P_h+P_c)+P_h \leqslant \tau \leqslant (n+1)(P_h+P_c)$; $n \gg 1$)

Cold gas flow periods are governed by the following equations.

The packing and wall to gas local heat flow equation is

$$-(\dot{m}c_p)_c V\frac{\partial T_c(x,\tau)}{\partial x}+\bar{\rho}Vc_s\frac{\partial T_s(x,\tau)}{\partial\tau}+\alpha_{i,c}A_i[T_c(x,\tau)-T_w(x,\tau,R)]=0 \quad (13)$$

with the cold gas entering at

$$T_c(L,\tau)=T_{c,in}. \quad (14)$$

Convective heat transfer from the packing to the gas is generally different in these periods so that

$$\bar{\rho}Vc_s\frac{\partial T_s(x,\tau)}{\partial\tau}+\alpha_c A[T_s(x,\tau)-T_c(x,\tau)]=0. \quad (15)$$

So is the gas to wall heat transfer and in these periods, instead of equation (7), a boundary condition of the form

$$-\lambda_w\frac{\partial T_w(x,\tau,R)}{\partial r}+\alpha_{i,c}[T_w(x,\tau,R)-T_c(x,\tau)]=0 \quad (16)$$

should be associated to equation (5).

Accordingly

$$-\lambda_e\frac{\partial T_e(x,\tau,R+\delta_w+\delta_e)}{\partial r}+\alpha_{o,c}[T_\infty-T_e(x,\tau,R+\delta_w+\delta_e)]=0 \quad (17)$$

should be considered, instead of equation (11), for conduction equation (9).

Next, the procedure to obtain the overall (macro) balance equation of the regenerator from the set of microbalance equations is described.

(i) Integrate equation (9) over the insulation volume

$$\int_{R+\delta_w}^{R+\delta_w+\delta_e} r\,\mathrm{d}r.$$

(ii) Make use of equation (10) in the equation obtained under (i).

(iii) For the hot period make use of equation (11) in the equation obtained under (ii).

(iv) For the cold period make use of equation (17) in the equation obtained under (ii).

(v) Integrate equation (5) over the wall volume

$$\int_R^{R+\delta_w} r\,\mathrm{d}r.$$

(vi) For the hot period make use of equation (7) in the equation obtained under (v).

(vii) For the cold period make use of equation (16) in the equation obtained under (v).

(viii) Add the equations obtained under (iii) and (vi) to eliminate $(R+\delta_w)\lambda_w\partial T_w(x,\tau,R+\delta_w)/\partial r$.

(ix) Add the equations obtained under (iv) and (vii) to eliminate $(R+\delta_w)\lambda_w\partial T_w(x,\tau,R+\delta_w)/\partial r$.

(x) Integrate equations obtained in (viii) and (ix) over the bed length

$$\int_0^L \mathrm{d}x.$$

(xi) For the hot period integrate the equation obtained in (x) with respect to time from $n(P_h+P_c)$ to $n(P_h+P_c)+P_h$.

(xii) For the cold period integrate the equation obtained in (x) with respect to time from $n(P_h+P_c)+P_h$ to $(n+1)(P_h+P_c)$.

(xiii) Add the equations obtained in (xi) and (xii) and apply the wall and insulation cyclic equilibrium conditions, equations (8) and (21), respectively.

(xiv) Define the average external insulation surface temperature in the hot and cold period

$$\bar{T}_{e,surf,h}=\frac{1}{LP_h}\int_0^L\int_{n(P_h+P_c)}^{n(P_h+P_c)+P_h} T_e(x,\tau,R+\delta_w+\delta_e)\,\mathrm{d}\tau\,\mathrm{d}x \quad (18)$$

and

$$\bar{T}_{e,surf,c}=\frac{1}{LP_c}\int_0^L\int_{n(P_h+P_c)+P_h}^{(n+1)(P_h+P_c)} T_e(x,\tau,R+\delta_w+\delta_e)\,\mathrm{d}\tau\,\mathrm{d}x \quad (19)$$

respectively, and rewrite the overall wall and insulation heat balance equation obtained in (xiii) in the form

$$R\left\{\alpha_{i,h}\int_0^L\int_{n(P_h+P_c)}^{n(P_h+P_c)+P_h}[T_h(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x+\alpha_{i,c}\int_0^L\int_{n(P_h+P_c)+P_h}^{(n+1)(P_h+P_c)}[T_c(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x\right\}$$
$$=(R+\delta_w+\delta_e)L[P_h\alpha_{o,h}(\bar{T}_{e,surf,h}-T_\infty)+P_c\alpha_{o,c}(\bar{T}_{e,surf,c}-T_\infty)]. \quad (20a)$$

Remark 1. Since $A_i=2R\pi L$ and $A_e=2(R+\delta_w+\delta_e)\pi L$, it is appropriate to have equation (20a) written in the form

$$\frac{A_i}{L}\left\{\alpha_{i,h}\int_0^L\int_{n(P_h+P_c)}^{n(P_h+P_c)+P_h}[T_h(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x+\alpha_{i,c}\int_0^L\int_{n(P_h+P_c)+P_h}^{(n+1)(P_h+P_c)}[T_c(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x\right\}$$
$$=A_e[P_h\alpha_{o,h}(\bar{T}_{e,surf,h}-T_\infty)+P_c\alpha_{o,c}(\bar{T}_{e,surf,c}-T_\infty)]. \quad (20)$$

This equation states that the quantity of heat transferred from the gases to the surface of the inner wall in both the hot and cold period (one complete cycle) is equal to that transferred from the external insulation surface to the surroundings. (The logic of this result confirms the consistency of the part of the model being used for its derivation.)

(xv) Integrate the hot gas local heat balance equation, equation (1), over the bed length

$$\int_0^L \mathrm{d}x$$

and make use of the hot fluid inlet condition, equation (2).

(xvi) Integrate the cold gas local heat balance equation, equation (13), over the bed length

$$\int_0^L \mathrm{d}x$$

and make use of the cold fluid inlet condition, equation (14).

(xvii) For the hot period integrate the equation obtained in (xv) with respect to time from $n(P_h+P_c)$ to $n(P_h+P_c)+P_h$.

(xviii) For the cold period integrate the equation obtained in (xvi) with respect to time from $n(P_h+P_c)+P_h$ to $(n+1)(P_h+P_c)$.

(xix) Add the equations obtained in (xvii) and (xviii) and apply the packing cyclic equilibrium condition, equation (4).

(xx) Define the mean hot and cold gas outlet temperatures

$$T_{h,out} = \frac{1}{P_h}\int_{n(P_h+P_c)}^{n(P_h+P_c)+P_h} T_h(L,\tau)\,\mathrm{d}\tau \qquad (21)$$

and

$$T_{c,out} = \frac{1}{P_c}\int_{n(P_h+P_c)+P_h}^{(n+1)(P_h+P_c)} T_c(0,\tau)\,\mathrm{d}\tau \qquad (22)$$

respectively, and rewrite the overall gases and wall heat balance equation obtained in (xix) in the form

$$(\dot{m}c_pP)_h V(T_{h,in}-T_{h,out})-(\dot{m}c_pP)_c V(T_{c,out}-T_{c,in})$$
$$= A_i\left\{\alpha_{i,h}\int_0^L\int_{n(P_h+P_c)}^{n(P_h+P_c)+P_h}[T_h(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x\right.$$
$$\left.+\alpha_{i,c}\int_0^L\int_{n(P_h+P_c)+P_h}^{(n+1)(P_h+P_c)}[T_c(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x\right\}. \qquad (23a)$$

Remark 2. Since the bed volume is $V = A_f L$, where $A_f = R^2\pi$ is the frontal area, and the mass flow rate is $\dot{M} = \dot{m}A_f$, it is appropriate to have equation (23a) written in the form

$$(\dot{M}c_pP)_h(T_{h,in}-T_{h,out})-(\dot{M}c_pP)_c(T_{c,out}-T_{c,in})$$
$$= \frac{A_i}{L}\left\{\alpha_{i,h}\int_0^L\int_{n(P_h+P_c)}^{n(P_h+P_c)+P_h}[T_h(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x\right.$$
$$\left.+\alpha_{i,c}\int_0^L\int_{n(P_h+P_c)+P_h}^{(n+1)(P_h+P_c)}[T_c(x,\tau)-T_w(x,\tau,R)]\,\mathrm{d}\tau\,\mathrm{d}x\right\}. \qquad (23)$$

This equation states that the quantity of heat not being transferred from the hot gas to the cold gas during the complete cycle is transferred to the regenerator wall. (This is again a correct statement for the diabatic regenerator modelled as above.)

The final step is given below.

(xxi) Add equations (20) and (23) to eliminate the double integral terms (read: to eliminate the wall temperature field which is just an intermediator since the wall does not form the external boundary of the regenerator), and to obtain the overall energy balance equation of the diabatic regenerator

$$(\dot{M}c_pP)_h(T_{h,in}-T_{h,out})-A_eP_h\alpha_{o,h}(\bar{T}_{e,surf,h}-T_\infty)$$
$$= (\dot{M}c_pP)_c(T_{c,out}-T_{c,in})+A_eP_c\alpha_{o,c}(\bar{T}_{e,surf,c}-T_\infty). \qquad (24)$$

Statement narrative. The quantity of heat released by the hot gas minus the heat loss to the surroundings during the hot gas flow period is equal to the quantity of heat received by the cold gas plus the heat loss to the surroundings during the cold gas flow period at the cyclic equilibrium operation of a diabatic regenerator.

Note that each side of equation (24) quantitatively expresses the quantity of heat (Q_{act}) being actually transferred at the cyclic steady state operation of a diabatic regenerator. Being an overall balance, this equation connects just the quantities at the system boundaries. Also note that, besides the mean hot and cold gas outlet temperatures, $T_{h,out}$ and $T_{c,out}$, respectively, this equation imposes the knowledge of the average insulation surface temperatures ($\bar{T}_{e,surf,h}$ and $\bar{T}_{e,surf,c}$) in both periods. Unfortunately, none of the references listed in this article mention a need either to calculate them from the solution of the mathematical model or to have them under control in experimental measurements. This is the main reason for the misinterpretations being present in the works on diabatic regenerator performance. In the next section some consequences of this fact on the interpretation of the regenerator effectiveness are discussed.

DIABATIC REGENERATOR EFFECTIVENESS

The effectiveness of any two-fluid heat exchanger essentially is a dimensionless measure of the quantity of heat actually being transferred among two streams. It is a normalized (from zero to unity) actual quantity of heat. This normalization needs the recognition of the maximal possible fluid enthalpy change in the system. This hypothetical quantity of heat (Q_{max}) can be seen as the enthalpy change of the weak stream undergoing the maximal possible temperature change ($T_{h,in}-T_{c,in}$) without any losses. For regenerators by a 'weak stream', or more precisely a 'weak period', the one with the smaller of the two possible heat capacities, $(\dot{M}c_pP)_h$ and $(\dot{M}c_pP)_c$, should be understood. Thus, the attribution of 'min' should be given to the period for which the relation

$$(\dot{M}c_pP)_{\min} = \min\,[(\dot{M}c_pP)_{\rm h}, (\dot{M}c_pP)_{\rm c}] \tag{25}$$

holds, so that

$$Q_{\max} = (\dot{M}c_pP)_{\min}(T_{\rm h,in}-T_{\rm c,in}). \tag{26}$$

The regenerator effectiveness is then simply defined as

$$\varepsilon = \frac{Q_{\rm act}}{Q_{\max}} \tag{27}$$

and it is a unique measure of its thermal performance. By the uniqueness here it is meant that the same ε should be obtained by writing $Q_{\rm act}$ either in terms of hot period parameters (left-hand side of equation (24)) or in terms of cold period parameters (right-hand side of equation (24)). Thus, for the model under consideration, the diabatic regenerator effectiveness is

$$\varepsilon = \frac{(\dot{M}c_pP)_{\rm h}(T_{\rm h,in}-T_{\rm h,out})-A_{\rm e}P_{\rm h}\alpha_{\rm o,h}(\bar{T}_{\rm e,surf,h}-T_\infty)}{(\dot{M}c_pP)_{\min}(T_{\rm h,in}-T_{\rm c,in})} \tag{28}$$

$$= \frac{(\dot{M}c_pP)_{\rm c}(T_{\rm c,out}-T_{\rm c,in})+A_{\rm e}P_{\rm c}\alpha_{\rm o,c}(\bar{T}_{\rm e,surf,c}-T_\infty)}{(\dot{M}c_pP)_{\min}(T_{\rm h,in}-T_{\rm c,in})}. \tag{29}$$

In order to rewrite these equations in terms of usual regenerator parameters (utilization factors $(\Pi/\Lambda)_{\rm h} = (\dot{M}c_pP)_{\rm h}/(M_{\rm s}c_{\rm s})$ and $(\Pi/\Lambda)_{\rm c} = (\dot{M}c_pP)_{\rm c}/(M_{\rm s}c_{\rm s})$) divide both the denominators and the numerators by the packing total heat capacity $M_{\rm s}c_{\rm s} = \bar{\rho}Vc_{\rm s} = \bar{\rho}A_{\rm f}Lc_{\rm s}$. This will yield

$$\varepsilon = \frac{(\Pi/\Lambda)_{\rm h}[T_{\rm h,in}-T_{\rm h,out}-B_{\rm h}(\bar{T}_{\rm e,surf,h}-T_\infty)]}{(\Pi/\Lambda)_{\min}(T_{\rm h,in}-T_{\rm c,in})} \tag{30}$$

$$= \frac{(\Pi/\Lambda)_{\rm c}[T_{\rm c,out}-T_{\rm c,in}-B_{\rm c}(\bar{T}_{\rm e,surf,h}-T_\infty)]}{(\Pi/\Lambda)_{\min}(T_{\rm h,in}-T_{\rm c,in})} \tag{31}$$

where

$$B_{\rm h} = \frac{A_{\rm e}\alpha_{\rm o,h}}{(\dot{M}c_p)_{\rm h}} \tag{32}$$

and

$$B_{\rm c} = \frac{A_{\rm e}\alpha_{\rm o,c}}{(\dot{M}c_p)_{\rm c}} \tag{33}$$

are dimensionless heat transfer coefficients to the surroundings in hot and cold periods, respectively. The structure of these diabatic regenerator parameters makes it reasonable to term them as the 'number of heat loss units'.

The most obvious conclusion from equations (30) and (31) is that the *adiabatic* regenerator effectiveness is obtainable as a special case by letting both $B_{\rm h}$ and $B_{\rm c}$ be zero. However, we are primarily interested in the interpretation of equations (30) and (31) for a *diabatic* regenerator (non-zero values of $B_{\rm h}$ and $B_{\rm c}$). It is most elementary to put these two equations, by using equations (I) and (II), into the forms

$$\varepsilon = \varepsilon_{\rm h} - \Delta\varepsilon_{\rm h} \tag{34}$$

and

$$\varepsilon = \varepsilon_{\rm c} + \Delta\varepsilon_{\rm c} \tag{35}$$

where

$$\Delta\varepsilon_{\rm h} = \frac{(\Pi/\Lambda)_{\rm h}}{(\Pi/\Lambda)_{\min}}B_{\rm h}\frac{\bar{T}_{\rm e,surf,h}-T_\infty}{T_{\rm h,in}-T_{\rm c,in}} = \frac{A_{\rm e}P_{\rm h}\alpha_{\rm o,h}}{(\dot{M}c_pP)_{\min}}\frac{\bar{T}_{\rm e,surf,h}-T_\infty}{T_{\rm h,in}-T_{\rm c,in}} \tag{36}$$

and

$$\Delta\varepsilon_{\rm c} = \frac{(\Pi/\Lambda)_{\rm c}}{(\Pi/\Lambda)_{\min}}B_{\rm c}\frac{\bar{T}_{\rm e,surf,c}-T_\infty}{T_{\rm h,in}-T_{\rm c,in}} = \frac{A_{\rm e}P_{\rm c}\alpha_{\rm o,c}}{(\dot{M}c_pP)_{\min}}\frac{\bar{T}_{\rm e,surf,c}-T_\infty}{T_{\rm h,in}-T_{\rm c,in}} \tag{37}$$

are respectively the corrections of the 'hot' ($\varepsilon_{\rm h}$) and 'cold effectiveness' ($\varepsilon_{\rm c}$) as defined by equations (I) and (II). From equations (34) and (35) one immediately obtains the difference

$$\varepsilon_{\rm h}-\varepsilon_{\rm c} = \Delta\varepsilon_{\rm h}+\Delta\varepsilon_{\rm c} = \frac{(\Pi/\Lambda)_{\rm h}B_{\rm h}(\bar{T}_{\rm e,surf,h}-T_\infty)+(\Pi/\Lambda)_{\rm c}B_{\rm c}(\bar{T}_{\rm e,surf,c}-T_\infty)}{(\Pi/\Lambda)_{\min}(T_{\rm h,in}-T_{\rm c,in})} = \frac{A_{\rm e}[P_{\rm h}\alpha_{\rm o,h}(\bar{T}_{\rm e,surf,h}-T_\infty)+P_{\rm c}\alpha_{\rm o,c}(\bar{T}_{\rm e,surf,c}-T_\infty)]}{(\dot{M}c_pP)_{\min}(T_{\rm h,in}-T_{\rm c,in})}. \tag{38}$$

A trial to compile what has been written by various authors about the difference $\varepsilon_{\rm h}-\varepsilon_{\rm c}$ might be misleading, but one thing is evident: it has never been interpreted in the terms appearing on the right-hand side of equation (38). In a diabatic regenerator this difference never vanishes and the parameters featuring in equation (38) have to be under control.

Consider now the interpretation of $\varepsilon_{\rm ave} = (\varepsilon_{\rm h}+\varepsilon_{\rm c})/2$ being used by many authors as a characteristic of diabatic regenerator operation at the cyclic steady state. Again it is most elementary, by using the equations given above, to establish the following relation:

$$\varepsilon_{\rm ave} = \tfrac{1}{2}(\varepsilon_{\rm h}+\varepsilon_{\rm c}) = \varepsilon+\tfrac{1}{2}(\Delta\varepsilon_{\rm h}-\Delta\varepsilon_{\rm c})$$
$$= \varepsilon+\tfrac{1}{2}\times\frac{(\Pi/\Lambda)_{\rm h}B_{\rm h}(\bar{T}_{\rm e,surf,h}-T_\infty)-(\Pi/\Lambda)_{\rm c}B_{\rm c}(\bar{T}_{\rm e,surf,c}-T_\infty)}{(\Pi/\Lambda)_{\min}(T_{\rm h,in}-T_{\rm c,in})}$$
$$= \varepsilon+\tfrac{1}{2}\times\frac{A_{\rm e}[P_{\rm h}\alpha_{\rm o,h}(\bar{T}_{\rm e,surf,h}-T_\infty)-P_{\rm c}\alpha_{\rm o,c}(\bar{T}_{\rm e,surf,c}-T_\infty)]}{(\dot{M}c_pP)_{\min}(T_{\rm h,in}-T_{\rm c,in})}. \tag{39}$$

Obviously $\varepsilon_{\rm ave}$ overpredicts the actual diabatic regenerator effectiveness ε. An identity $\varepsilon_{\rm ave} \equiv \varepsilon$ is true if and only if the heat loss to the surroundings during the hot period is exactly the same as the heat loss during the cold period

$$A_e P_h \alpha_{o,h} (\bar{T}_{e,surf,h} - T_\infty) = A_e P_c \alpha_{o,c} (\bar{T}_{e,surf,c} - T_\infty). \quad (40)$$

This equality cannot be verified unless each of its terms is known. Consequently ε_{ave} as it was used in some works loses its justification.

CONCLUDING REMARKS

A consistent diabatic regenerator performance evaluation must include the same parameters as for an adiabatic case and additionally the average external surface temperatures and heat transfer coefficients at this surface in both periods, as well as the ambient temperature. Prediction of these heat transfer coefficients might be satisfactory if empirical correlations are used, but control of the space and time averaged surface temperatures will require sophisticated experimental techniques. The results presented in refs. [1–8] should be reconsidered in accordance with the facts being outlined in this paper.

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INTERPRETATIONS ERRONEES DES PERFORMANCES DU GENERATEUR DIABATIQUE

Résumé—On présente les sources de désaccord entre les efficacités "chaude" (ε_h) et "froide" (ε_c) d'un régénérateur diabatique. Le modèle de paroi à conductivité finie est utilisé pour évaluer la différence entre ε_h et ε_c. On dégage un besoin de mesures expérimentales des températures moyennes de surface externe de l'isolant dans les deux périodes.

FEHLINTERPRETATION DES WIRKUNGSGRADES VON VERLUSTBEHAFTETEN REGENERATOREN

Zusammenfassung—In dieser Arbeit werden die Ursachen für den Unterschied bei der Berechnung des Wirkungsgrades eines verlustbehafteten Regenerators (a) auf der warmen Seite (ε_h), (b) auf der kalten Seite (ε_c) vorgestellt. Zur Bestimmung dieses Unterschiedes wird die Regeneratorwand modellhaft nachgebildet. Für beide Perioden werden dringend Meßdaten bezüglich der mittleren Temperatur der äußeren Oberfläche der Wärmedämmung benötigt.

ОБ ОШИБОЧНОЙ ТРАКТОВКЕ ХАРАКТЕРИСТИК ДИАБАТИЧЕСКОГО РЕГЕНЕРАТОРА

Аннотация—Показано коренное различие между "горячей" (ε_h) и "холодной" (ε_c) эффективностью диабатического регенератора. Модель регенератора с конечной проводимостью стенки используется для вывода разности между ε_h и ε_c. Подчеркивается необходимость экспериментального измерения средней температуры внешней изолированной поверхности в обоих случаях.